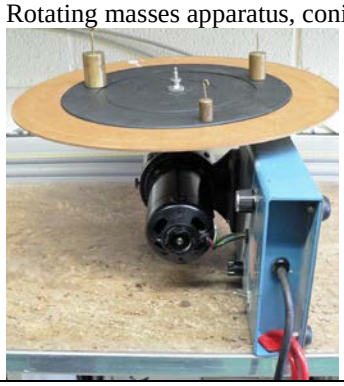
Circular Motion by Inquiry

1	Going in Circles		Buggies, string, hover pucks, scale-pendulum apparatus
2	Centripetal Acceleration	Notes: 122-126 Problems: Pg. 126 #8, 10 Simulation: Ladybug Motion 2D Video: Circular Motion Idea Video: Circular Motion Idea 2	Complete: *Going in Circles* Start: *Centripetal Acceleration*
3	Centripetal Acceleration *continued*	Notes: pg. 128-133 Problems: Pg. 133 #6, Pg. 138 #7 Simulation: Ladybug Revolution Video: Tension in Vertical Circle	Complete: *Centripetal Acceleration*
4	Thinking About Circular Motion	Homework: *Thinking About Circular Motion* Video: Conical Pendulum Video: Frames of Reference	Rotating masses apparatus, conical pendulum 
5	CGPS or experiment		The Circular Motion Challenge: Rubber stopper, string, pen tube, alligator clips, balance scale Circular Motion Experiment: student choice of equipment

SPH4U: Going in Circles?

What makes the world go ‘round? Let’s find out!

Recorder: _____
Manager: _____
Speaker: _____
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A: Observing Circular Motion

Let’s find out how forces in different directions affect the motion of the intrepid physics buggy! A reminder: **velocity** has two parts: a magnitude (speed) and a direction.

1. Describe the velocity (speed and direction) of the buggy when it drives along the level floor with no additional forces.

2. Attach a piece of string to the buggy – if it tends to flip, adjust the string angle. In the chart below, describe how the velocity (speed and direction) of the buggy changes when you exert a constant, horizontal force on the buggy in a direction: (a) parallel to its motion, and (b) perpendicular to the direction of the buggy’s motion.

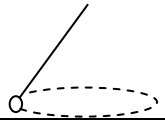
Forces Parallel	Forces Perpendicular

3. In which case(s) above is the buggy accelerating? Explain.

4. Make the buggy move in a circle with the string horizontal. Draw a FBD for the buggy from a head-on point of view at two different moments in time (as if you were lying on the ground at a moment when it was traveling directly towards or away from you).

a) Which objects interact with the buggy? b) What forces or components of forces balance? c) What is the direction of the net force experienced by the buggy?	FDB	FBD

5. Tie an object to a length of string and swing it in a **slow**, **horizontal** circle. (Don’t hit anyone!)



a) Which objects interact with the circling object? b) What forces or components of forces balance? c) What is the direction of the net force experienced by the object?	FDB (head-on view)

B: Making Rules and Breaking Rules

1. Devise a provisional rule for the direction of the net force of an object moving in a circle at a constant speed.

Provisional Rule #1:

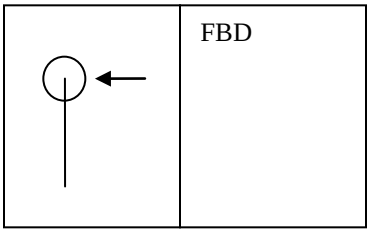
2. From your observations or from your own experience when you move in a circle you may have noticed other characteristics of forces in circular motion. Here are some other plausible provisional rules for circular motion:

Provisional Rule #2: *When an object moves in a circle there is a force acting outwards (away from the centre)*

Provisional Rule #3: *When an object moves in a circle, there must be a force "forwards", in the direction the object is moving.*

We don't know yet which of these rules are correct. In science, we make observations, develop theories (rules) and then test them out. In the following two experiments we will look for evidence which supports or refutes these three rules.

3. **Hover puck and string**. A hover puck is attached to a string. The hover puck is given a short push and is released. The end of the string is held fixed.
(a) Predict how the puck will move **after** the push.



(b) Test your prediction and describe the motion of the puck.

(c) Draw a FBD for the puck **after the push** from a bird's eye view (from above).

(d) Each rule makes a prediction about this situation (after the push) which is shown below. Based on your observations and your understanding of the forces, which rules are supported and which are refuted? If you think a force is present, you must explain what real force it might be.

Rule #1	Rule #2	Rule #3
Prediction: There is a net force pointing towards the centre of the circular path.	**Prediction**: There is a force in the outwards direction (away from the centre)	**Prediction**: There is a force in the forwards direction (tangent to the circular path).
Conclusion:		

4. **Hover puck released!** A hover puck is initially moving in a circle due to a string. The end of the string is released.
(a) Try this out and describe the motion of the puck after it is released.

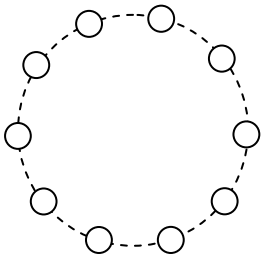
(b) The only force that changed when the string is released is the inwards force of tension. Based on your observations and your understanding of the forces, which rules are supported and which are refuted? Explain briefly

Rule #1	Rule #2	Rule #3
Prediction: If the net force is now zero, it will travel in a straight line.	**Prediction**: If there is an outwards force, it is still there, so it will travel directly outwards.	**Prediction**: If there is a forwards force, it is still there, so it should accelerate away.
Conclusion:		

5. Elevate one of the provisional rules to an official rule. Mention what uniform circular motion means.

Net Force and Uniform Circular Motion:

6. **Prediction**. Use your new force rule to explain how to cause the hover puck to move in a circle simply by using gentle taps from a metre stick. Illustrate the force of your taps in the diagram. As a class, we will test this out (move on for now).



Centripetal is an adjective that describes any force or component of a force that points towards the centre of curvature of an object's path of motion (the centre of a circle). A centripetal force changes an object's direction without changing its speed. It is responsible for keeping an object moving in a circle. There is no special symbol for a centripetal force – **never write F_c, anywhere**! Centripetal is simply a new category for already familiar forces (F_t, F_g, F_n, etc.) and is a helpful term when discussing circular motion.

7. Which forces in the examples we have investigated so far are centripetal forces?

8. (*as a class*) Watch the simulation for the car travelling around a corner. As a passenger in the car, explain what is happening from the earth frame when we:
(a) slide across the seat

(b) are "pressed" against the car door

D: The Pendulum of Fate

In our classroom we have a 0.50 kg mass hanging on the end of a 1.0 m string. The string is connected to a spring scale.

1. The mass is at rest, hanging from the scale. Explain how you can decide what the spring scale will read. Predict the spring scale reading and support this with the appropriate illustrations and equations.

2. Imagine you pull the mass to the side and release it such that it swings like a pendulum and moves along **a circular path**. Even though the speed of the pendulum changes, at the bottom of the swing our new rule still applies (think of the FBD at that moment). When the ball passes directly under the scale, will the scale reading be greater than, less than or equal to the reading at rest. Use the new force rule to come up with your prediction.

3. Perform the experiment. Record the outcome and compare it to your prediction. Does it support or refute the force rule? Explain.

Adapted from *The Physics Active Learning Guide* by A. Van Heuvelen, Pearson, 2006

SPH4U: Centripetal Acceleration

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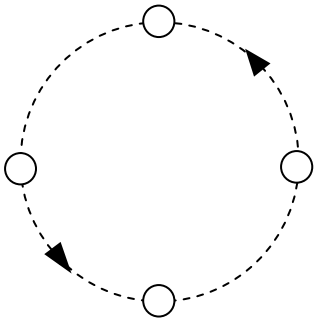
What is special about the motion of an object moving in a circle? Only vectors will tell!

A: Instantaneous Velocity and a Curving Path

Think back to our experiments in the previous investigation with the hover puck and string. In one case, we let go of the string and observed the motion of the hover puck. This gives us a hint about the instantaneous velocity of the puck at the moment the string was released.

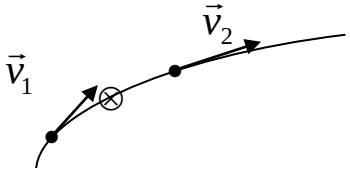


1. Draw an instantaneous velocity vector for the hover puck at the four moments in time shown in the diagram. When drawing instantaneous velocity vectors, they should start at the object and appear **tangent** to an object's path.

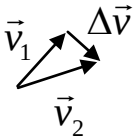
2. When an object moves in a circle at a steady rate, does the speed change? Does the velocity change? Explain.

B: Acceleration

The technique explained below will help you estimate the average acceleration of an object during two-dimensional motion. In grade 11 you learned that adding vectors is done by drawing them tip-to-tail. To find the acceleration, we need to find a change in velocity, $v_2 - v_1$. Subtracting two vectors is done by drawing them **tail to tail.**



We want to estimate the acceleration for an object when it passes the middle point (⊗). Draw the initial velocity vector (v_1) just before the point. Draw the final velocity vector (v_2) just after the point.

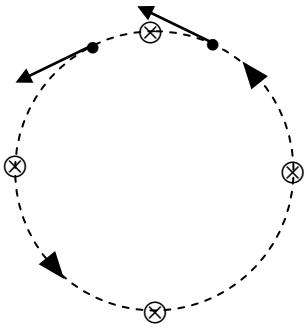


Redraw the velocity vectors tail to tail. Draw the change in velocity vector going from the head of v_1 to the head of v_2.

The acceleration, a, is in the same direction as the change in velocity, Δv, and has a magnitude $\Delta v/\Delta t$.

C: Acceleration and Circular Motion

1. An object moves at a constant speed in a circle. Use the vector subtraction technique to estimate the acceleration vector at the four moments shown in the diagram. **Neatly** show your vector work for each example (the first example is almost complete). Draw a wiggly acceleration vector inside the circle for each moment.

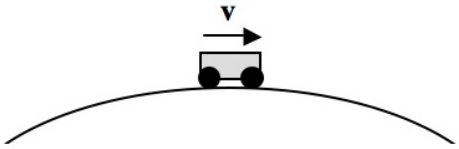


2. What is the pattern to the acceleration vectors found in the previous question?

An object moving in a circle experiences a radial or **centripetal acceleration, $\mathbf{a_c}$,** which points towards the centre of its circular path. In uniform circular motion, the magnitude of the centripetal acceleration is constant and the direction is always changing.

3. How does the pattern for the acceleration vectors compare with the rule you developed for the net force in the previous investigation? What law of physics is this a result of?

4. A physics buggy moves at a constant speed across the top of a circular hill, as shown in the illustration. In the chart below, use the vector subtraction technique to find the acceleration, draw a FBD, and explain whether the two results agree according to Newton's 2nd law.

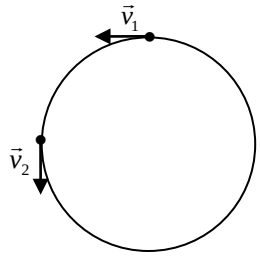
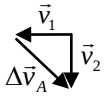
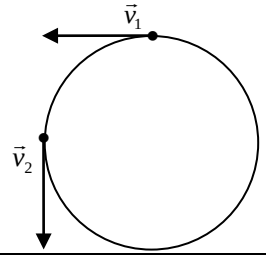
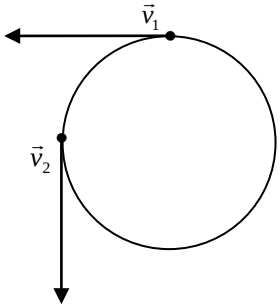


Vectors	FBD	Agreement?

5. Have you ever been in a situation like the one described above? What was the situation and what did it *feel* like?

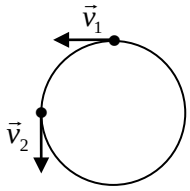
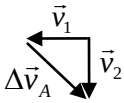
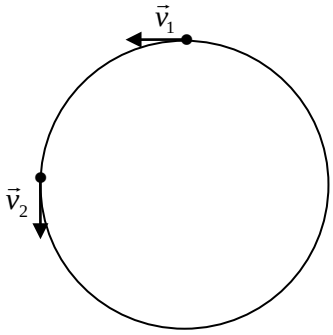
D: Acceleration, Speed and Radius

1. Three different physics buggies travel with a constant speed along three identical circular paths. Buggy A moves with speed v, Buggy B with speed $2v$ and Buggy C with speed $3v$. Buggy A completes a ¼ trip in a time Δt_A and experiences a change in velocity Δv_A. Use the vector subtraction technique to compare the centripetal accelerations of the three buggies.

	Illustration	Change in Velocity	Time Interval	Acceleration	Summary
Buggy A			For a ¼ trip: Δt_A	$\vec{a}_A = \frac{\Delta\vec{v}_A}{\Delta t_A}$	How does the magnitude of the acceleration depend on the speed? Write out a mathematical expression for this relationship.
Buggy B		*Hint*: How does Δv_B compare with Δv_A?	How does Δt_B compare with Δt_A?		
Buggy C					

Adapted from *The Physics Active Learning Guide* by A. Van Heuvelen, Pearson, 2006

2. Two identical buggies travel with the same constant speed along two circular paths with different radii. Buggy A moves in a circle of radius r and buggy B moves in a circle with radius $2r$. Buggy A completes a ¼ trip in a time Δt_A. Use the vector subtraction technique to compare the centripetal acceleration of each buggy.

	Illustration	Change in Velocity	Time Interval	Acceleration	Summary
Buggy A			Δt_A	$\vec{a}_A = \dfrac{\Delta\vec{v}_A}{\Delta t_A}$	How does the magnitude of the acceleration depend on the radius? Write out a mathematical expression for this relationship.
Buggy B					

3. Combine the two results above and write a general expression for the magnitude of the centripetal acceleration.

E: Test the Acceleration Expression

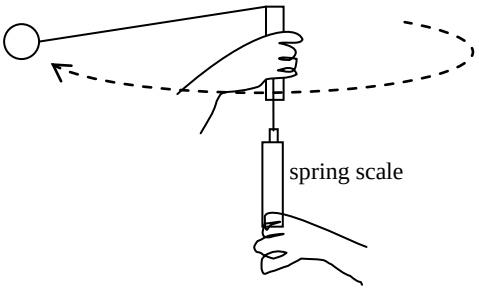
There are two other ways of writing your general expression for the centripetal acceleration: $a_c = 4\pi^2 r f^2 = 4\pi^2 r/T^2$ where f is the frequency and T is the period of the circular motion. Recall that $f = 1/T$.

1. Show that these are mathematically equivalent to your expression assuming an object moves in a complete circle (a distance of $2\pi r$ in a time T).

2. According to our understanding of centripetal acceleration and net force, explain what happens to the magnitude of the centripetal force if the object spins faster with the same radius.

3. Explain what happens to the magnitude of the centripetal force if the object spins with the same **frequency** and the radius increases.

4. Find a small tube, string (at least 1 m), rubber stopper, and spring scale (5 N). Attach the string to the stopper and thread it through the tube. Have one person hold the tube and another person hold a spring scale connected to the bottom of the string. Find some space where you can swing the stopper in a horizontal circle in the air above your heads. Don't hit people! Practice swinging it at a steady rate.



5. Test your two predictions above. Note that keeping the same frequency is quite tricky! Were your predictions consistent with the observations?

SPH4U: Thinking About Circular Motion

Recorder: _____
Manager: _____
Speaker: _____
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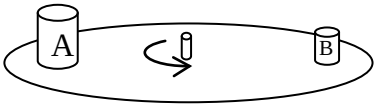
A: Testing Our Understanding of Circular Motion

Two objects of mass 100 g and 200 g that have identical bottom surfaces are placed on a rotating platform that can turn with increasing speed. The two masses are placed at equal distances from the centre and have equal speeds at each moment in time.



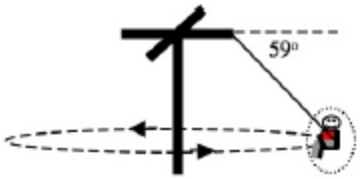
1. Complete the chart below. The coefficient of static friction is μ_s.

FBD	*y*-component of 2nd law	*x*-component of 2nd law	**Prediction**: Which mass will fly off first, or do they fly off at the same time? (Hint: what is the maximum speed for each before it slips?)
Object A			
Object B			

2. Was your prediction consistent with the outcome of the experiment? Explain. What might have caused any discrepancies? (Hint: Think of the assumptions you made.)

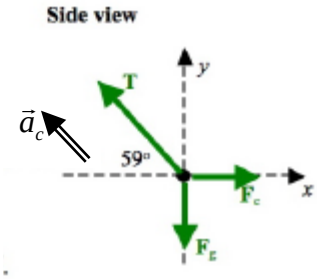
B: Evaluate the Problem

Identify any errors in the solution to the following problem. Provide a corrected solution if there are errors.



Problem: 80-kg Samuel rides at a constant 6.0-m/s speed in a horizontal 6.0-m radius circle in a seat at the end of a cable that makes a 59° angle with the horizontal. Determine the tension in the cable.

Proposed Solution: The situation is pictured above. We simplify by assuming that Samuel, the system, is a particle. A FBD for Samuel is shown at the right along with the acceleration direction.

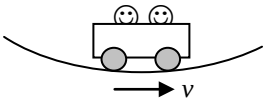
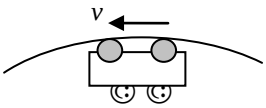
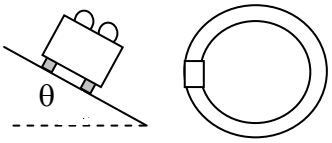


$$F_c = m(v^2/r)$$
$$= (80\text{ kg})(6.0\text{ m/s})^2/(6.0\text{ m})$$
$$= 480\text{ N}$$

The tension is 480 N.

C: Representations of Circular Motion

Complete the chart for situations #1-3. For #4 and 5, you must come up with a possible situation that agrees with the equation. Then complete the chart and write a description as if it were a textbook problem – be creative!

Words and Sketch	Direction of a_c (vector subtraction technique)	FBD	Newton's 2nd Law
(1) A roller coaster car moves along a frictionless circular dip in the track. 			
(2) The roller coaster moves past the top of a frictionless loop-the-loop. 			
(3) The rollercoaster travels around a banked corner on a frictionless track. At this moment it is travelling directly **out of the page**. 	*Hint*: Draw from a top view	*Hint*: Draw from a head-on view. There are no forces parallel to the incline	
(4)			900 N – (50 kg)(9.8 N/kg) = (50 kg)v^2 / (12 m)
(5)			200 N + (50 kg)(9.8 N/kg) = (50 kg)v^2 / (12 m)

SPH4U: Experimenting with Circular Motion

Recorder: _____
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Speaker: _____
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A: Experimental Ideas

Your task is to design a method to determine the magnitude of the net force exerted on the mass of a **conical pendulum** as the mass moves at a constant speed in a circle of a chosen radius. *Hint*: for more accuracy, use a circle with a large radius. You may use any of our classroom equipment.

1. (10 minutes) On your whiteboard, quickly outline an experiment that would allow you to measure the net force experienced by a conical pendulum. For ideas on what to consider, look at the list of headings in part B below.

2. Present to the class your experimental proposal.

B: Experimental Design

1. Based on ideas from the presentations, write a careful description of the method you will use to determine the net force. Draw a labeled sketch.

Method & Sketch

2. Draw a FBD

3. Write the physical quantities you will measure and the physical quantities you will calculate.

measured:

calculated:

4. Write the mathematical procedure you will use to determine the net force.

5. List additional assumptions.

6. List sources of experimental uncertainty and how to minimize them.

7. Perform the experiments and compare the two results. Discuss how assumptions and experimental uncertainties contribute to the discrepancy between the outcomes of the two experiments.

CGPS: The Circular Motion Challenge!

Do not write on this page!

Materials
Small tube
String
Rubber stopper
Mystery object
Stop watch
Alligator Clips

Spin the stopper in a horizontal circle with a mystery object (~ 50 – 150 g) attached below. Make any measurements you think are reasonable to determine the mass of the object hanging from the string at the bottom of the tube. The one measurement you **cannot** make is directly measuring the mass of your mystery object!

Hint: Use the alligator clips to make sure the radius of the swinging stopper remains constant!

Hint: It works best when it swings fairly quickly so the string is **almost** horizontal. In that case, you may make reasonable assumptions based on the fact that the angle between the string and the horizontal is very small.

Warning: Don't hit people with the swinging stopper!

Problem:

Recorder: _____
Manager: _____
Speaker: _____
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A. The Picture

Diagrams, important quantities, measurements, coordinate system, what happens?

B. The Question

Precise physics question, label quantities to find

C. The Plan

Why? Explain the physics relationships	Key steps	"Textbook" equations and

D. The Work

Specific equations, algebraic work (no number crunching!), verify units, explanations

E. The Results

Substitute values, state answer to question

Reasonable size, units, direction?

SPH4U: How Noteworthy!

An important part of our course is the taking of good quality notes from your textbook. This is meant to help you solidify what you have learned during that day's lesson. Your notes will be randomly checked for quality and completion, and are expected to be completed by the following class. You syllabus lists all the notes you are expected to take. For example, today your syllabus says:

Kinematics

1	Short quiz on Introduction Taking Notes	Gr. 11 Review Questions: 1-D Kinematics Gr. 11 Review Lessons: Kinematics Notes: pg. 18-23

Physics Course Notes

A: Read

Read the entire section first – beginning to end. Don't skip the boxes in the margins or the sample problems! Always ask the question, "OK, but why?" Read the pages listed above. Now.

B: Key Ideas

Identify what you think the key physics ideas are. These are usually **concepts**, such as "force and mass affect an object's acceleration" or "in the absence of air resistance, all objects fall at the same rate of acceleration. In point form, briefly note below what key ideas you came across in your reading. Whenever possible, **write in your own words**.

C: Definitions

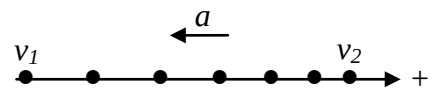
Record the definitions of important physics ideas or quantities. This should be both verbal and mathematical whenever possible.

D: Examples

Our goal when taking notes is to avoid direct copying, which requires very little thinking. When your notes are carefully organized, it is easy to refer back to your textbook for any lengthy passages that you might be tempted to copy out. So don't! Instead, explain the **logical process** of the solutions

For example, consider Sample problem 2, pg. 19. Your summary might go like this:

> We know the initial velocity ($\vec{v}_1$), average acceleration ($\vec{a}$) of the motorcycle and the time while it brakes (Δt). Since the motorcycle is slowing down, we can use the definition of acceleration $\left(\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}\right)$, rearrange it, and solve for the final velocity ($\vec{v}_2$).



The description is complete if a typical student would be able to **understand** and solve the problem by following your instructions, given the actual values. A more complicated problem may involve a few more statements, but would read in a similar way. Please sketch a **motion diagram** or a **free body diagram**, whenever appropriate.

E: Format

A consistent and clear format will help you organize your notes. Please follow this guide:

Sample Notes

Unit: Kinematics
Topic: Acceleration in One Dimension (pg. 18-23)

Idea:
Definition:
Examples:
} Repeat as necessary

Homework problems:

F: Homework Problems

It is often helpful to include your homework problems with the notes themselves. To earn full marks, homework must be shown using the GRASP procedure (see tomorrow's investigation for details.)

G: Passing Notes!

On a separate sheet of note paper, use the format above to take notes from pages 18-23. This may take about 20 minutes for you to do. Your goal is a note that is neat, complete and concise so that it will be a useful study aid. When you have completed you notes, pass your note and this paper to your neighbour and they will mark it!

H: Evaluating Notes

	Neighbour 1	
Follows format, organized and neat	5 4 3 2 1 0	Messy, disorganized, format missing
Key ideas well explained with appropriate definitions	5 4 3 2 1 0	Key ideas missed or poorly explained, definitions lacking
Examples carefully summarized with clear instructions, diagrams	5 4 3 2 1 0	Examples poorly explained and missing key steps
Overall, a clear note that would be valuable to study from	5 4 3 2 1 0	Very unhelpful - four months from now you will have no idea what this note is about

Suggestions, if any, to improve: _____

	Neighbour 2	
Follows format, organized and neat	5 4 3 2 1 0	Messy, disorganized, format missing
Key ideas well explained with appropriate definitions	5 4 3 2 1 0	Key ideas missed or poorly explained, definitions lacking
Examples carefully summarized with clear instructions, diagram	5 4 3 2 1 0	Examples poorly explained and missing key steps
Overall, a clear note that would be valuable to study from	5 4 3 2 1 0	Very unhelpful - four months from now you will have no idea what this note is about

Suggestions, if any, to improve: _____